Math 32B, Lecture 4
Multivariable Calculus

Sample Final Exam

Instructions: You have three hours to complete the exam. There are ten problems, worth a total of one hundred points. You may not use any books, notes, or calculators. Show all your work; partial credit will be given for progress toward correct solutions, but unsupported correct answers will not receive credit. Remember to make your drawings large and clear, and to label your axes.

Write your solutions in the space below the questions. If you need more space, use the back of the page. Do not turn in your scratch paper.

Name: ______________________________

UID: ______________________________

Section: ______________________________

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
Total:	80	

Problem 1.

Let

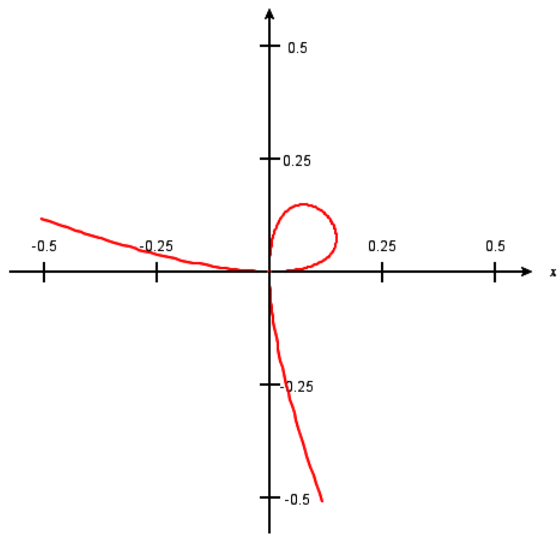
$$p(x,y) = \begin{cases} Cxy \text{ if } 0 \le x \le 1 \text{ and } 0 \le y \le 1-x \\ 0 \quad \text{otherwise} \end{cases}$$

(a) [5pts.] Find a constant C that makes $p(x,y)$ into a probability distribution.

(b) [5pts.] Find $P(X \ge Y)$.

Problem 2.

Let $\mathbf{r}(t) = \langle t^2(1-t), t(t-1)^2 \rangle$. A plot of $\mathbf{r}(t)$ is shown below.

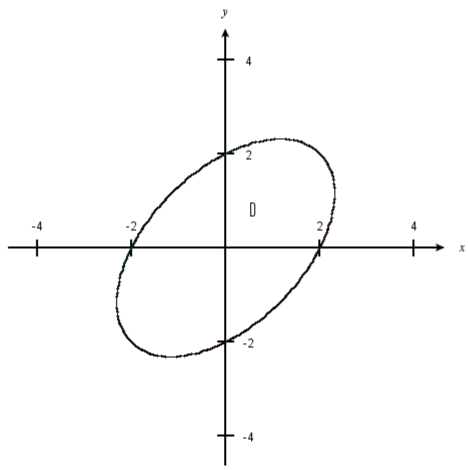


(a) [5pts.] Compute the area enclosed by the loop in the curve.

(b) [5pts.] What is the flux of the vector field $\mathbf{F}(x, y) = \langle 2x - 7y^2, 9x - 2y \rangle$ out of the loop?

Problem 3.

Consider the domain $\mathcal{D}$ shown below, which consists of the points x, y such that $x^2 - xy + y^2 \leq 4$.



(a) [5pts.] Suppose that $G(u, v) = \left(2u - \frac{2}{\sqrt{3}}v, 2u + \frac{2}{\sqrt{3}}v\right)$. Find the region in the uv-plane that maps to $\mathcal{D}$ under G.

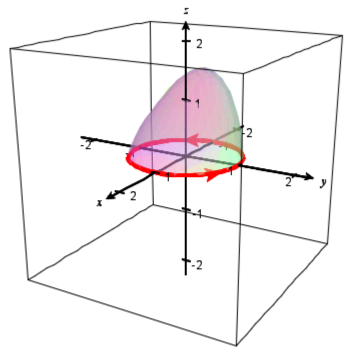
(b) [5pts.] What is $\int\int_{\mathcal{D}} (x^2 - xy + y^2)dA$?

Problem 4.

Consider the vector field $\mathbf{F} = \langle 2ye^z - xy, y, yz - z \rangle$.

(a) [5pts.] Verify that $\mathbf{A} = \langle yz, xyz, y^2e^z \rangle$ is a vector potential for $\mathbf{F}$.

(b) [5pts.] What is the flux of $\mathbf{F}$ across the surface shown? The marked curve is the boundary of the surface.



Problem 5.

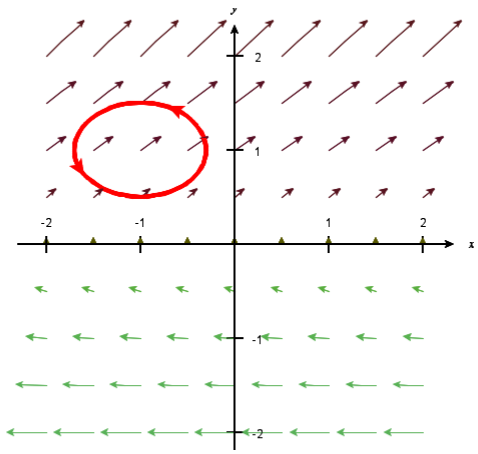
Consider the vector field $\mathbf{F}(x, y) = \langle 9y - y^3, e^{\sqrt{y}}(x^2 - 3x)\rangle$. Let $\mathcal{C}$ be the square with corners $(0, 0)$, $(0, 3)$, $(3, 3)$, and $(0, 3)$, oriented counterclockwise.

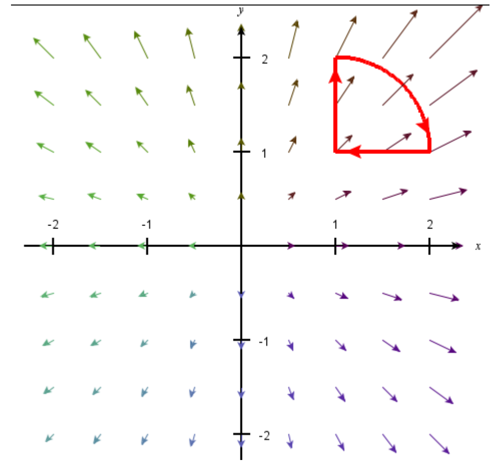
(a) [5pts.] Show that $\mathbf{F}$ is not conservative.

(b) [5pts.] What is $\oint_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}$? [Hint: What does $\mathbf{F}$ look like on $\mathcal{C}$?]

Problem 6.

Consider the vector fields and paths shown below. The path on the left is $\mathcal{C}_1$ and the path on the right is $\mathcal{C}_2$.





(a) [5pts.] For each vector field above, decide whether curl($\mathbf{F}$) is positive, negative, or zero at the origin. Justify your answers.

(b) [5pts.] Decide whether the line integrals $\oint_{\mathcal{C}_1} \mathbf{F}\cdot d\mathbf{r}$ and $\oint_{\mathcal{C}_2} \mathbf{F}\cdot d\mathbf{r}$ are positive, negative, or zero. Justify your answers.

Problem 7.

Let $\mathcal{S}$ be the boundary of the solid bounded by the cylinder $x^2 + y^2 = 16$ and the planes $z = 0$ and $z = 4 - y$, oriented outward.

(a) [5pts.] Draw this surface.

(b) [5pts.] Find the flux of the vector field $\mathbf{F}(x, y, z) = \langle xyz + xy, \frac{1}{2}y^2(1 - z) + e^x, e^{x^2+y^2} \rangle$ through $\mathcal{S}$.

Problem 8.

Consider the surface $z = x^2 + y^2$ with $0 \leq z \leq 4$, with the outward pointing normal vector.

(a) [3pts.] Draw this surface. Be sure to orient the boundary.

(b) [3pts.] Is the flux of $\mathbf{F}(x, y, z) = \langle 2x, 0, -7z^2 \rangle$ across $\mathcal{S}$ positive or negative? Justify your answer.

(c) [4pts.] Find the surface area of $\mathcal{S}$. Use any method you like.